Defining Selection Functions

HOLGER ANDREAS AND MARIO GÜNTHER

Abstract: In AGM-style belief revision theory, various types of epistemic ordering have been described. First, we argue that the notion of an epistemic ordering on a finite belief base is cognitively most plausible and most suitable for the analysis of specific examples. Second, we show how other types of epistemic ordering can be derived from an epistemic ordering on a finite belief base.

Keywords: belief revision, epistemic ordering, partial meet contraction

1 Introduction

The notion of an epistemic ordering is essential to AGM belief revision theory, founded by Alchourrón, Gärdenfors, and Makinson (1985). Various types of such an ordering have been described by the authors of the AGM theory and others working in the AGM framework. Each type gives rise to at least one distinct belief revision scheme. In essence, we can distinguish between the following types of epistemic ordering:

1. ordering on a finite belief base H
2. ordering on the union of the remainder sets of a finite belief base H
3. ordering on a logically closed belief set K
4. ordering on the union of the remainder sets of a logically closed belief set K
5. plausibility ordering on possible worlds.

Arguably, the notion of an epistemic ordering on a finite belief base H is cognitively the most plausible type of ordering. When we come to analyse concrete examples of belief changes, it is by far easier to specify an epistemic ordering on a finite belief base than on an infinite belief set. Directly specifying an epistemic ordering on a logically closed belief set is, strictly speaking, beyond the cognitive capacities of a finitely bounded human mind since such a set is an infinite entity. Hence, it is desirable to have a means for determining an epistemic ordering on logically closed belief set on the basis of an epistemic ordering on a finite belief base.

Similar considerations apply to the problem of directly specifying an epistemic ordering on the union of the remainder sets of a finite belief base. (The remainder set $H \perp \alpha$ is the set of a maximal subsets of H that do not entail α; see Section 3 for the precise definition.) Even though the members of such a set are finite entities, it is difficult to form intuitions about how entrenched a finite set of beliefs is when taken as a whole unit. While this is not a problem for proving representation theorems about belief changes, it turns out to be a problem when we come to work out concrete applications of belief revision theory, including the analysis of conditionals, causal relations, and scientific explanations. In this context, recall that a cognitively more plausible analysis of conditionals was a key motivation for Gärdenfors to study belief changes.[1] Hence, considerations of cognitive plausibility and applicability should matter for our theories of belief revision.

Let us, finally, discuss epistemic orderings of type (4) and (5). If directly specifying an epistemic ordering on a logically closed belief set is beyond the cognitive means of a finitely bounded human mind, then the direct specification of an epistemic ordering on the union of remainder sets of a logically closed belief set is a fortiori so. Possible worlds furnished with a plausibility ordering have great intuitive appeal. At the same time, we think that the plausibility of a world depends on which sentences are true in this world, but not the other way around. We do not have intuitions about the plausibility of a possible world taken as a whole unit, but about sentences that hold true in this world. For example, we say that possible worlds in which presently presumed laws of physics hold true are more plausible than others in which some of these laws do not hold. Notice, moreover, that Lewis (1979) specifies some guidelines about the similarity ordering of possible worlds in terms of differences among possible worlds at the level of singular facts and laws.

In sum, we can say that the notion of an epistemic ordering on a finite belief base has a relatively well defined, direct empirical content, in the sense that some of our explicit beliefs are more firmly established than others. Other types of epistemic ordering seem to be more theoretical, in the sense of being less directly related to our explicit beliefs. It is therefore

[1] See the foreword by David Makinson of the 2008 reprint of Gärdenfors (1988) by College Publications, edited by Dov Gabbay.

desirable to investigate if we can derive, respectively, an epistemic ordering of the types (2) to (5) from an epistemic ordering on a finite belief base, thereby relating certain theoretical concepts of belief revision theory to concepts with a more direct empirical content. This is the problem to which the present paper is addressed.

Another motivation for pursuing the present investigation is that belief revision schemes with an epistemic ordering on a finite belief base have received relatively little attention in the belief revision literature. As observed by Hansson (1999, p. 103), it proved difficult to define entrenchment-based revisions for belief bases in a manner closely analogous to entrenchment-based belief set revisions. The theory of ensconcement relations by Williams (1994) is, arguably, too radical a proposal because the corresponding revision operation "cuts away" unnecessarily many beliefs. The preferred subtheory approach by Brewka (1991) is simple and intuitive, but does not satisfy all postulates of belief base revision established by Hansson (1993), and so deviates more strongly from the original AGM theory than Hansson's seminal account of belief base revision in (1993). The latter account, however, works with selection functions, which in turn are based on an epistemic ordering on the union of the remainder sets of a belief base.

The overall strategy for deriving epistemic orderings of type (2) to (4) from an epistemic ordering of type (1) is to define selection functions of appropriate type on the basis of a strict weak ordering on a finite belief base. This directly gives us an epistemic ordering of type (2) and (4). An epistemic entrenchment ordering on a logically closed belief set can be defined via partial meet contractions, which in turn are based on selection functions. A plausibility ordering on a set of possible worlds will be defined in terms of degrees of satisfaction of a finite belief base with priorities.

2 Levels of Epistemic Priority

Let H be a finite belief base. That is, H is a set of sentences that are intended to represent explicit beliefs. Drawing on the work of Brewka (1991), we assume the epistemic ordering among the items of H to be a strict weak ordering. Such an ordering can be represented by a sequence of subsets of H:

$$\mathbf{H} = \langle H_1, \ldots, H_n \rangle.$$

where $H_1, \ldots, H_n$ is a partition of H. $\mathbf{H}$ is called a *a prioritised belief base*. The indices represent an epistemic ranking of the beliefs. H_1 is the set of the most firmly established beliefs, the beliefs in H_2 have secondary priority, etc. In other words, partitioning H into disjoint subsets is intended to introduce different levels of epistemic priority into the "flat" belief base H.

It must be acknowledged that the assumption of a strict weak ordering on H is still an idealization. For some pairs of explicit beliefs, we may lack an intuition as to which belief is more firmly established, and it may not always be appropriate to infer from this that the two beliefs are equally firm. Partial orderings of a belief base are considered in Brewka (1991), but in this paper we shall not address the problem of deriving epistemic orderings on the basis of a partially ordered belief base.

3 Partial Meet Revision

Hansson (1993) defines (internal) partial meet base revisions in a manner closely analogous to partial meet revisions in Alchourrón et al. (1985). Both belief revision schemes hinge on the notion of a selection function, defined for remainder sets of the belief base and the belief set, respectively. Drawing on the presentation in Hansson (1999, Ch. 1), we recall the core definitions underlying the two belief revision schemes. Let us begin with the notion of a *remainder set* of A relative to α, where A may or may not be finite.

Definition 1 $A \bot \alpha$
Let A be a set of sentences and α a sentence. $A' \in A \bot \alpha$ iff

1. $A' \subseteq A$
2. $\alpha \notin Cn(A')$
3. *there is no A'' such that $A' \subset A'' \subseteq A$ and $\alpha \notin Cn(A'')$.*

$Cn(A)$ designates the logical closure of A. Some members of a remainder set $A \bot \alpha$ may be epistemically superior to others, in the sense that their beliefs are more firmly held than the beliefs of other members of $A \bot \alpha$. At this point, the notion of a selection function comes into play. Suppose $\leq$ is a binary transitive relation on the union of all remainder sets $A \bot \alpha$. $B \leq B'$ means that B' is epistemically not inferior to B. Equivalently, $B \leq B'$ means that B' is epistemically at least as good as B. Using such an epistemic ordering, we can define a selection function for the remainder set as follows:

$$\sigma(A\bot\alpha) = \{A' \in A\bot\alpha \mid A'' \leq A' \text{ for all } A'' \in A\bot\alpha\}. \qquad \text{(Def } \sigma\text{)}$$

Then, we take the selected members of the remainder set to define the contraction of A by α:

$$A - \alpha = \bigcap \sigma(A\bot\alpha). \qquad \text{(PMC)}$$

Now we are in a position to define partial meet revisions using the Levi identity $(A * \alpha = (A - \alpha) + \alpha)$:

$$A * \alpha = \bigcap \sigma(A\bot\neg\alpha) + \alpha. \qquad \text{(PMR)}$$

In line with the presentation in Hansson (1999, Ch. 1), we have not made any assumption as to whether the set A of sentences is finite or not. Hence, A may be a finite belief base or a logically closed belief set. Note, however, that belief bases and belief sets differ as regards the definition of expansions, represented by the symbol $+$.

4 Defining Selection Functions

How can we determine a selection function based on some intuition about an epistemic ranking of sentences in $H \subseteq A$? The rationale of the following definitions is to select those members of $A\bot\,\alpha$ that overlap with the more firmly established beliefs in H to the greatest extent. Let us make this idea more precise:

Definition 2 *Let A be a set of sentences, and A'' and A' be subsets of A. $A'' < A'$ iff there is i $(1 \leq i \leq n)$ such that*

1. $A'' \cap H_i \subset A' \cap H_i$, *and*
2. *for all* $j < i$ $(j \geq 1), A'' \cap H_j = A' \cap H_j$.

Definition 3 $A' \equiv A''$ *iff* $A' \cap H_i = A'' \cap H_i$ *for all* i $(1 \leq i \leq n)$.

Definition 4 $A' \leq A''$ *iff* $A' < A''$ *or* $A' \equiv A''$.

Since $\leq$ is defined on $A \times A$, it is well defined for the union of all remainder sets of A.

In the classical AGM theory, it is desirable that the relation $\leq$ underlying the selection function σ is transitive. If this condition is satisfied, it can be shown that the revision function $*$ – defined by (Def σ), (PMC), and (PMR) – satisfies the postulates K^*1 to K^*8 (see Theorems 3.1 and 4.16 in Gärdenfors (1988)). As is well known, these postulates are considered canonical for belief set revisions in the AGM theory. For this reason, let us show that the relation $\leq$ defined by Definition 4 is in fact transitive, using the transitivity of the relation $<$ defined by Definition 2.

Proposition 1 *If $A''' < A''$ and $A'' < A'$, then $A''' < A'$.*

Proof. Suppose (i) $A''' < A''$ and (ii) $A'' < A'$. By Definition 2, there are i and l $(1 \leq i, l \leq n)$ such that

(i) $A''' \cap H_i \subset A'' \cap H_i$ and for all $j < i$ $(j \geq 1)$, $A''' \cap H_j = A'' \cap H_j$, and

(ii) $A'' \cap H_l \subset A' \cap H_l$ and for all $k < l$ $(k \geq 1)$, $A'' \cap H_k = A' \cap H_k$.

We consider now three exhaustive cases:

(a) $l = i$: Then, there is some i such that $A''' \cap H_i \subset A'' \cap H_i = A'' \cap H_l \subset A' \cap H_l$ and $A''' \cap H_k = A'' \cap H_k = A' \cap H_k$ for all $k < l$ $(k \geq 1)$. Because of the transitivity of the strict subset relation $\subset$ and Definition 2, it holds that $A''' < A'$, as desired.

(b) $l > i$: Then, there is some i such that $A''' \cap H_i \subset A'' \cap H_i = A' \cap H_i$ and $A''' \cap H_j = A'' \cap H_j = A' \cap H_j$ for all $j < i$ $(j \geq 1)$. By the transitivity of the strict subset relation $\subset$ and Definition 2, we can infer from this that $A''' < A'$, as desired.

(c) $l < i$: Then, there is some l such that $A''' \cap H_l = A'' \cap H_l$, but $A'' \cap H_l \subset A' \cap H_l$. Hence, $A''' \cap H_l \subset A' \cap H_l$. From $l < i$ we can furthermore infer that $A''' \cap H_k = A'' \cap H_k = A' \cap H_k$ for all $k < l$. By Definition 2, these two conclusions imply that $A''' < A'$, as desired.

Thus, we have shown that $<$ defined by Definition 2 is transitive. □

We show now that $\leq$ defined by Definition 4 is transitive as well.

Proposition 2 *If $A''' \leq A''$ and $A'' \leq A'$, then $A''' \leq A'$.*

Proof. Assume $A''' \leq A''$ and $A'' \leq A'$. By Definition 4, this implies $A''' < A''$ or $A''' \equiv A''$, and $A'' < A'$ or $A'' \equiv A'$. We need to consider four cases.

(a) Suppose $A''' < A''$ and $A'' < A'$. Then, by Proposition 1, $A''' < A'$. By Definition 4, $A''' \leq A'$.

(b) Suppose $A''' < A''$ and $A'' \equiv A'$. Then, by Definition 2, there is i $(1 \leq i \leq n)$ such that (i) $A''' \cap H_i \subset A'' \cap H_i$ and (ii) for all $j < i$ $(j \geq 1)$, $A''' \cap H_j = A'' \cap H_j$. By Definition 3, $A'' \equiv A'$ implies that, for all k $(1 \leq k \leq n)$, $A'' \cap H_k = A' \cap H_k$. Using (i) and (ii), we can infer from this that (iii) $A''' \cap H_i \subset A' \cap H_i$ and (iv) for all $j < i$ $(j \geq 1)$, $A''' \cap H_j = A' \cap H_j$. By Definition 4, $A''' \leq A'$.

(c) Case (c) is identical to case (b), if we simply switch $<$ and $\equiv$ such that the supposition is $A''' \equiv A''$ and $A'' < A'$.

(d) Suppose $A''' \equiv A''$ and $A'' \equiv A'$. Then, by Definition 3, for all i $(1 \leq i \leq n)$, $A''' \cap H_i = A'' \cap H_i$ and $A'' \cap H_i = A' \cap H_i$. Hence, by the transitivity of equality $=$ on sets, $A''' \cap H_i = A' \cap H_i$ for all i $(1 \leq i \leq n)$. By Definition 4, we obtain $A''' \equiv A'$, and thus $A''' \leq A'$, as desired.

Thus, we have shown that $\leq$ defined by Definition 4 is transitive. □

5 Epistemic Entrenchment

Entrenchment-based revisions are at the core of the classical AGM theory. Let us begin with the formal characterisation of the epistemic entrenchment relation. $\alpha \leq_e \beta$ means that α is at most as entrenched as β. The following postulates formally characterise this relation (Gärdenfors, 1988, Ch. 4.6):

$$\text{If } \alpha \leq_e \beta \text{ and } \beta \leq_e \chi, \text{ then } \alpha \leq_e \chi \tag{EE1}$$

$$\text{If } \alpha \vdash \beta, \text{ then } \alpha \leq_e \beta \tag{EE2}$$

$$\alpha \leq_e \alpha \wedge \beta \text{ or } \beta \leq_e \alpha \wedge \beta \tag{EE3}$$

$$\text{When } K \neq K_\perp, \alpha \notin K \text{ iff } \alpha \leq_e \beta \text{ for all } \beta \in K \tag{EE4}$$

$$\text{If } \beta \leq_e \alpha \text{ for all } \beta \in \mathcal{L}, \text{ then } \alpha \in Cn(\emptyset). \tag{EE5}$$

where $\mathcal{L}$ is the set of all formulas of the formal language used to analyse belief changes, and $K_\perp$ the absurd belief set containing all elements of $\mathcal{L}$.

As shown by Gärdenfors and Makinson (1988), epistemic entrenchment orderings and contractions are interdefinable by the following equations:

$$\begin{aligned} &\beta \in K - \alpha \text{ iff } \beta \in K \\ &\qquad \text{and either } \alpha < (\alpha \vee \beta) \text{ or } \alpha \in Cn(\emptyset). \end{aligned} \tag{G-}$$

$$\beta \leq_e \alpha \text{ iff } \beta \notin K - (\alpha \wedge \beta) \text{ or } \alpha \wedge \beta \in Cn(\emptyset). \qquad (\text{C}\leq)$$

The idea underlying (C≤) is that a contraction of K with $\alpha \wedge \beta$ results in the retraction of α or β (or both), and β should be retracted iff α is at least as epistemically entrenched as β. Note that, according to (C≤), $\beta < \alpha$ means (in the non-tautological case) that $\alpha \in K - (\alpha \wedge \beta)$.

Note that an ordering of epistemic entrenchment must satisfy certain logical constraints that may not hold for the epistemic ranking of a prioritised belief base $\mathbf{H}$. For this reason, we cannot simply extend the epistemic ranking of a prioritised belief base $\mathbf{H}$ in order to obtain an epistemic entrenchment ordering. But we can determine an epistemic entrenchment ordering using (C≤) and partial meet contractions defined on the basis of a selection function σ, which in turn is determined by a prioritised belief base $\mathbf{H}$ via Definition 4. To be more precise:

Proposition 3 *Let $-$ be a contraction function defined on the basis of a prioritised belief base $\mathbf{H}$, via definitions (Def σ), (PMC) and 4. Further, let $\leq_e$ be defined using this contraction function via C≤. $\leq_e$ thus defined satisfies the postulates $(EE1) - (EE5)$ of a standard relation of epistemic entrenchment.*

This proposition follows from theorems 4.16 and 4.30 in Gärdenfors (1988).

6 Plausibility Ordering of Possible Worlds

The epistemic orderings considered so far are syntactic in nature since they are defined on sets or sets of sets of sentences. However, an epistemic ordering can also be represented by a system of spheres of possible worlds, as shown by Grove (1988) and Spohn (1988). The basic ideas of this representation are as follows. The possible worlds of the innermost sphere are considered most plausible in the epistemic state to be modelled. The further away a possible world is from the innermost sphere, the less plausible this world is considered.

To describe the formal properties of such a system of spheres, a few more symbols need to be introduced. If a α is a sentence, $[\alpha]$ designates the set of possible worlds that verify α. If A is a set of sentences, let $[A]$ designate the set of possible worlds that verify all members of A. $Sent(w)$, by contrast, is the set of sentences that are verified by the possible world w.

Now we are in a position to define the notion of a system of spheres along the lines of Grove (1988):

Definition 5 *Let W be a set of possible worlds, $\mathcal{L}$ a propositional language, and let K be a belief set. Further, let $<_p$ be a binary relation on W. $\langle W, <_p \rangle$ is a system of spheres centered on K iff*

(S1) $\leq$ *is a strict, weak order on W, i.e., asymmetric, transitive, and modular*

(S2) $[K] = min_{<_p}(W)$

(S3) $[T] = W$, *where T is a tautology*

(S4) $min_{<_p}(W \cap [\alpha])$ *is well defined for all sentences α of $\mathcal{L}$.*

$min_{<_p}(W')$ designates the set of $<_p$-minimal elements in a set $W' \subseteq W$ of possible worlds. In more formal terms:

$$min_{<_p}(W') = \{w \mid \text{there is no } w' \text{ such that } w' <_p w\}$$

$min_{<_p}(W \cap [\alpha])$ is the set of possible worlds considered most plausible after a revision by α. This suggests the following definition of revisions:

$$K * \alpha = \{\beta \mid \ w \models \beta \text{ for all } w \in min_{<_p}(W \cap [\alpha])\}.$$

Drawing on Benferhat, Dubois, Lang, and Prade (1993), we can define a system of spheres on the basis of a prioritised belief base:

Definition 6 *Let $\mathbf{H}$ be a finite prioritised belief base, and $\mathcal{L}$ a propositional language in which the sentences of $\mathbf{H}$ are given. Let W be a set of possible worlds such that $[T] = W$. $w <_p w'$ iff there is i $(1 \leq i \leq n)$ such that*

1. $|Sent(w') \cap H_i| < |Sent(w) \cap H_i|$, *and*
2. *for all* $j < i$ $(j \geq 1), |Sent(w') \cap H_j| = |Sent(w) \cap H_j|$.

The idea underlying this definition is that a possible world w is more plausible than another w' iff w satisfies more sentences at a certain epistemic level i of $\mathbf{H}$ (in the sense of satisfying a set $H_i' \subseteq H_i$ of sentences with a greater cardinality), while being on a par with w' at the epistemic levels $j < i$.

Proposition 4 *The relation $<_p$ (defined by Definition 6) is asymmetric, transitive, and modular.*

Proof. (1) Asymmetry. The proof is trivial and can be obtained using that the relation $<$ on cardinal numbers is asymmetric.

(2) Transitivity. The proof is analogous to that of Proposition 1, and can be obtained using the fact that the relation $<$ on cardinal numbers is transitive.

(3) Modularity. Modularity is also referred to as the property of being negatively transitive: if, $w' \not< w''$, $w'' \not< w'$, $w'' \not< w'''$, and $w''' \not< w''$, then $w' \not< w'''$ and $w''' \not< w'$. That is, the relation of being incomparable (according to $<_p$) is transitive. Suppose (i) $w' \not< w''$ and $w'' \not< w'$, and (ii) $w'' \not< w'''$ and $w''' \not< w''$. (i) implies (iii) that, at any level j $(1 \leq j \leq n)$, $|Sent(w') \cap H_j| = |Sent(w'') \cap H_j|$. (ii) implies (iv) that, at any level j $(1 \leq j \leq n)$, $|Sent(w'') \cap H_j| = |Sent(w''') \cap H_j|$. (iii) and (iv) imply that, at any level j $(1 \leq j \leq n)$, $|Sent(w') \cap H_j| = |Sent(w''') \cap H_j|$. Hence, $w' \not< w'''$ and $w''' \not< w'$. □

Proposition 5 *The relation $<_p$ (defined by Definition 6) has a well-defined minimum on any set $W' \subseteq W$.*

Proof. Suppose, for contradiction, that $<_p$ has not a well defined minimum on $W' \subseteq W$. By transitivity of $<_p$, this implies that there is an infinite sequence $w_i, w_{i+1}, w_{i+2}, \ldots$ such that $w_i < w_j$ if $i > j$. Hence, for any world w of this infinite sequence, there is another world w' that satisfies, at some epistemic level k, more sentences of H_k than w (in the sense of satisfying a set of sentences with a greater cardinality), while being on a par with w at the levels $l < k$. This implies that **H** has at least one component that is an infinite set. Contradiction. □

From propositions 4 and 5, we can infer that $<_p$ defines a system of spheres of possible worlds (in the sense of Definition 5), which is centred on $[K] = min_{<_p}(W)$.

Let us briefly indicate why we cannot define a system of spheres of possible worlds using a definition of $<_p$ that is closely analogous to Definition 2 (which defines an ordering on sets of sentences). Suppose we define $w <_p w'$ iff there is i $(1 \leq i \leq n)$ such that

1. $Sent(w') \cap H_i \subset Sent(w) \cap H_i$, and
2. for all $j < i$ $(j \geq 1), Sent(w') \cap H_j = Sent(w) \cap H_j$.

The order thus defined is not always modular, and so qualifies only as a strict partial order. Here is a simple counterexample to modularity. $\mathbf{H} = \langle\{s, p, q, r\}\rangle$. Let us represent possible worlds w by sets of literals such that, for any literal l, l or its complement is a member of w. $w' = \{p, r, \neg q, \neg s\}$, $w'' = \{p, q, \neg r, \neg s\}$, and $w''' = \{p, s, r, \neg q\}$. Then, we have $w' \not< w''$, $w'' \not< w'$, $w'' \not< w'''$, and $w''' \not< w''$, but $w''' < w'$.

7 Conclusion

AGM-style belief revision theory comes with a variety of epistemic orderings, each of which gives rise to at least one specific belief revision scheme. Of these types of epistemic ordering, the notion of an epistemic ordering on a finite belief base is most closely and most directly related to our intuitions about some beliefs being firmly established than others. Other types of epistemic ordering are comparable to theoretical concepts in scientific theories in at least two respects. First, they are less directly related to our intuitions about some beliefs being more entrenched than others. Second, they are motivated by elements of the respective belief revision scheme to a greater extent than the notion of an epistemic ordering on a finite belief base is. For example, the need for an epistemic ordering on a logically closed belief set arises with K^*1, which says that belief sets are logically closed. Hence, it is desirable to investigate if the more theoretical epistemic orderings can be determined on the basis of an epistemic ordering one a finite belief base. We have answered this question in the affirmative. To be more precise, we have shown how an epistemic ordering on unions of remainder sets, an epistemic entrenchment and a plausibility ordering can be determined on the basis of the epistemic ranking of a finite belief base.

References

Alchourrón, M. A., Gärdenfors, P., & Makinson, D. (1985). On the logic of theory change: Partial meet contraction functions and their associated revision functions. *Journal of Symbolic Logic*, *50*, 510–530.

Benferhat, S., Dubois, D., Lang, J., & Prade, H. (1993). Inconsistency management and prioritised syntax-based entailment. In *Proceedings of the 14th International Joint Conference on Artificial Intelligence (IJCAI-93)* (pp. 640–645). Los Altos: Morgan Kaufmann.

Brewka, G. (1991). Belief revision in a framework for default reasoning. In *Proceedings of the Workshop on The Logic of Theory Change* (pp. 602–622). London: Springer.

Gärdenfors, P. (1988). *Knowledge in Flux*. Cambridge, Mass.: MIT Press.

Gärdenfors, P., & Makinson, D. (1988). Revision of knowledge systems using epistemic entrenchment. In M. Vardi (Ed.), *TARK' 88 - Proceedings of the Second Conference on Theoretical Aspects of Reasoning about Knowledge* (pp. 83–95). Los Altos: Morgan and Kaufmann.

Grove, A. (1988). Two modellings for theory change. *Journal of Philosophical Logic*, *17*, 157-170.

Hansson, S. O. (1993). Reversing the levi identity. *Journal of Philosophical Logic*, *22*(6).

Hansson, S. O. (1999). *A Textbook of Belief Dynamics. Theory Change and Database Updating*. Dordrecht: Kluwer.

Lewis, D. (1979). Counterfactual dependence and time's arrow. *Noûs*, *13*(4), pp. 455-476.

Spohn, W. (1988). Ordinal conditional functions: A dynamic theory of epistemic states. In *Causation in Decision, Belief Change, and Statistics II* (pp. 105–134). Dordrecht: Kluwer.

Williams, M.-A. (1994). On the logic of theory base change. In C. MacNish, D. Pearce, & L. M. Pereira (Eds.), *Logics in Artificial Intelligence* (pp. 86–105). Berlin, Heidelberg: Springer.

Holger Andreas
University of British Columbia
Canada
E-mail: holger.andreas@ubc.ca

Mario Günther
Universität Regensburg
Germany
E-mail: mario.guenther@ur.de